

Exclusive decay of P -wave Bottomonium into double J/ψ

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Abstract

We calculate the relativistic corrections of J/ψ , including electromagnetic corrections, to $\chi_{bJ} \rightarrow J/\psi J/\psi$ in the framework of nonrelativistic QCD factorization. The relativistic effect is found to increase the lower-order prediction for the decay width by about 10%, while the electromagnetism contribution is very small, about 0.2% for χ_{b0} and χ_{b2} . The total branching ratio is predicted to be of order 10^{-5} for $\chi_{b0,b2} \rightarrow J/\psi J/\psi$, but 10^{-11} for $\chi_{b1} \rightarrow J/\psi J/\psi$, since there is only electromagnetism contribution in this channel. We predict it is possible to observe these reactions in LHC. Finally, we estimate the decay width and branching ratio of $\chi_{cJ} \rightarrow \omega\omega, \phi\phi$ in the constituent quark model by our formula at the leading-order of relativistic correction and electromagnetic correction. The obtained branch ratio of $\chi_{c0,2} \rightarrow \phi\phi$ is in agreement with the experimental measurement in the order of magnitude.

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I. INTRODUCTION

The nonrelativistic nature of heavy quarkonium provides people with a valid component to understand the nonrelativistic effect of QCD. Since the quarkonium includes several well-separated scales which contain both hard scale and soft scale, the studies of hadronic reactions involving heavy quarkonium are significative of giving us an insight into both perturbative and nonperturbative QCD.

The running of B factories with high luminosity has made the measurement of hadronic-exclusive processes feasible and given the results that raised new challenges to the existing theory. One of the largest puzzles is the cross section for exclusive double charmonium-production in annihilation i.e. $e^+e^- \rightarrow J/\psi + \eta_c$, at the B factory energy of $\sqrt{s} = 10.6$ GeV(Ref. [1–3]) is about an order of magnitude larger than the leading-order non-relativistic QCD(NRQCD) predictions in Refs. [4–6].

The NRQCD factorization [7] as an outstanding effective field theory approach to dealing with the physical problem involving multiple scales provides us with a systematic framework to deal with the exclusive quarkonium production process; the amplitude can be factorized as the products of the short-distance, perturbatively calculable coefficients and the long-distance, nonperturbative NRQCD matrix elements which are universal for all processes. The short-distance coefficient can be expanded by the order in α_s , while the NRQCD matrix elements are organized as a series in the relative velocity v of the heavy quark, so one can improve the NRQCD predictions simultaneously in α_s and v . Considering the next-to-leading-order perturbative corrections to $e^+e^- \rightarrow J/\psi + \eta_c$, the discrepancy between theory and experiment measurement is greatly alleviated [8–13].

In recent years, the studies of double-charmonium production at B factory have already obtained huge theoretical progress in understanding hadronic exclusive processes with charmonium production. These facts inspire us to explore more analogous and valuable processes to add to our knowledge about these exclusive processes involving quarkonium. Similar to e^+e^- annihilation, double-charmonium production in bottomonium decays can also be used to study the dynamics of hard exclusive processes and the structure of charmonium mesons; the reason is that the bottomonium can decay into not only light hadron but also charmonium because its mass is heavier than charmonium and close to the energy of B factory. Although the investigations about bottomonium decay are not as intense as double-charmonium production in annihilation, there are several works focusing on these decays [14–22]. In Ref.[17–22], the double-charmonium production in exclusive bottomonium decays $\eta_b \rightarrow J/\psi J/\psi$ has been investigated. These researches groped the potential of the discovery of this hadronic decay channel in experimentation. As the S -wave bottomonium decays into double J/ψ have been researched, it is natural to carry on a further study of the similar P -wave bottomonium decays into double J/ψ .

This paper is intended to deal with the P -wave bottomonium decay process $\chi_{bJ} \rightarrow J/\psi J/\psi$, which is a part of the processes dealt with in Refs.[14, 23]. In Ref.[14], light-cone method is employed, and only the QCD process is considered. However, using NRQCD factorization to handle this type of process with the initial and final states all involving heavy quarkonium is more natural. In this paper, we calculate the relativistic corrections to this process in the NRQCD frame; the pure QED process is also considered. In fact, this work is an extended and updated version of Ref. [24].

This paper is organized as follows. In Sec. II, we describe the NRQCD factorization formula relevant to this work and compare our matching scheme with the orthodox doctrine.

We also present a detailed description on how to determine the short-distance coefficients through relative order v^2 in $\chi_{bJ} \rightarrow J/\psi J/\psi$. In Sec. III, we perform calculations of the amplitudes for $\chi_{bJ} \rightarrow J/\psi J/\psi$ that include the relativistic corrections to the order of v^2 of J/ψ , helicity amplitudes and electromagnetism contributions. In Sec. IV, we apply our formulas to investigate the phenomenological impact of QCD and QED corrections to the decay width and branching ratio of $\chi_{bJ} \rightarrow J/\psi J/\psi$. By analyzing the numerical results, We predict the possibility of the observation of $\chi_{bJ} \rightarrow J/\psi J/\psi$ decay in the experimentation. Finally, we summarize our results in Sec. V.

II. NRQCD FACTORIZATION AND MATCHING STRATEGY

Heavy quarkonium is a nonrelativistic system involving multiple scales. Because the velocity v of the heavy quark is much less than 1, the following scales are well separated: the heavy-quark mass m_Q , the relative momentum $m_Q v$, and the binding energy $m_Q v^2$. NRQCD factorization provides us with a valid effective field theory to separate the scale m_Q from the others. In the NRQCD factorization formula, the contribution from the scale that is less than m_Q is kept in the Lagrange operators, while the contribution from the scale that is more than m_Q is absorbed in the Wilson coefficients. With NRQCD factorization, the decay width can be expressed as the sum over $q\bar{q}$ channels of products of a long-distance-physics-insensitive short-distance coefficient and a process independent long-distance non-perturbative matrix element.

For a $Q(p)\bar{Q}(\bar{p})$ pair with total momentum P and relative momentum q we express the momenta of c and \bar{c} in perturbative matching:

$$p = \frac{P}{2} + q, \quad \bar{p} = \frac{P}{2} - q, \quad (1)$$

where q and P satisfy $P_i \cdot q_i = 0$, and $p^2 = \bar{p}^2 = m_Q^2$. P is the true total momentum of the $c\bar{c}$ pair, $P = p + \bar{p}$, with invariant mass of $2E_q$. In the rest frame of the $c\bar{c}$ pair, the explicit components of the momenta are $P^\mu = (2E_q, \mathbf{0})$, $q^\mu = (0, \mathbf{q})$, $p^\mu = (E_q, \mathbf{q})$, and $\bar{p}^\mu = (E_q, -\mathbf{q})$, respectively.

To be accurate to order v^2 , there are two methods that can be used to match the short-distance coefficients. One is the traditional matching method, in which we need to expand the energy of the Q or the \bar{Q} in the $Q\bar{Q}$ rest frame E_q around the pole mass m_c in power series of \mathbf{q}^2 ,

$$E_q = m_c + \frac{\mathbf{q}^2}{2m_c} + \mathcal{O}(\mathbf{q}^4). \quad (2)$$

The other method which we used in this paper is to expand every occurrence of m_c in the amplitude in terms of \mathbf{q}^2/E_q^2 , while keeping E_q intact:

$$m_c = E_q - \frac{\mathbf{q}^2}{2E_q} + \mathcal{O}(\mathbf{q}^4). \quad (3)$$

In the first method, when summing the polarization states of $c\bar{c}({}^3S_1)$, $b\bar{b}({}^3P_J)$, there are new factors of E_q , which are regenerated in the squared amplitude, we have to reexpand these occurring E_q factors again and realign the corresponding terms from the leading order (LO) matrix element squared to the relativistic correction piece. The second method avoids many

complications that emerged in the first one and eliminates the task of matching the cross section to the amplitude squared.

There is another question that needs to be considered. In our matching method, m_c has been eliminated in favor of E_q in the physical matrix element squared, then we need decide which value of E_q should be taken to give the prediction. After comparing the Eq. (2) which comes from simple nonrelativistic kinematics and the G-K relation [25]:

$$\frac{M_{J/\psi}}{2m_c} = 1 + \frac{1}{2}\langle v^2 \rangle_{J/\psi} + O(v^4), \quad (4)$$

where $\langle v^2 \rangle_{J/\psi}$ is a dimensionless ratio of the vacuum matrix elements defined as follow:

$$\langle v^2 \rangle_{J/\psi} \approx \frac{\langle J/\psi(\lambda) | \psi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda) \chi | 0 \rangle}{m_c^2 \langle J/\psi(\lambda) | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda) \chi | 0 \rangle}. \quad (5)$$

We can see that theoretical consistency requires that E_q can be fixed in an unambiguous manner, i.e. E_q appearing everywhere in the short-distance coefficients can be replace by $M_{J/\psi}/2$. By this way, the relativistic effects in phase space integrals are automatically incorporated. In addition, choosing $M_{J/\psi}$ as the input parameter is better than m_c since the mass of J/ψ is known rather precisely while the charm quark mass is ambiguously defined.

Since we no longer need to worry about the complication from the phase space integral and sum of the polarization states, we can match the short-distance coefficients at the amplitude level. Because they are similar for the $\chi_{b0,b1,b2} \rightarrow J/\psi J/\psi$, here we take the χ_{b0} as an example to demonstrate how to match the short-distance coefficients. At the leading order of α_s and the order of v^2 , the amplitude $\mathcal{M}[\chi_{b0} \rightarrow J/\psi J/\psi]$ in terms of the vacuum-to- J/ψ and χ_{b0} -to-vacuum matrix elements can be written as:

$$\begin{aligned} \mathcal{M}_{\chi_{b0}} = & \sqrt{2M_{\chi_{b0}}} \sqrt{2M_{J/\psi_1}} \sqrt{2M_{J/\psi_2}} \sum_{m,n=0}^{m,n=1} c_{mn}^0 \langle J/\psi_1 | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^{2m} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_1) \chi | 0 \rangle \\ & \langle J/\psi_2 | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^{2n} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_2) \chi | 0 \rangle \frac{1}{\sqrt{3}} \langle 0 | \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma} \right) \psi | \chi_{b0} \rangle, \end{aligned} \quad (6)$$

where c_{mn}^0 are the corresponding short-distance coefficients, which are Lorentz scalars formed by various kinematic invariants in the reaction. In particular, they also depend explicitly on the helicity λ of J/ψ . For the Lorentz-invariant amplitude in the left-hand side of Eq. (6), $\mathcal{M}_{\chi_{b0}}$, it is most natural to assume relativistic normalization for each particle state, since the squared amplitude needs to be folded with the relativistic phase space integral to obtain the physical decay width. However, in the right-hand side of Eq. (6), the J/ψ and χ_{bJ} state appearing in the NRQCD matrix elements conventionally assume the nonrelativistic normalization. To compensate this difference, one must insert a factor $\sqrt{2M_{J/\psi}} \sqrt{2M_{J/\psi}} \sqrt{2M_{\chi_{bJ}}}$ in the right side of Eq. (6).

To determine the coefficients c_{mn}^0 we follow the moral that these short-distance coefficients are insensitive to the long-distance confinement effects, so one can replace the physical J/ψ state by a free $c\bar{c}$ pair of quantum number 3S_1 , and replace the physical χ_{bJ} state by a free $b\bar{b}$ pair of quantum number 3P_J , by which the NRQCD operator matrix elements can be perturbatively calculated. The short-distance coefficients c_{mn}^J can then be read off by comparing the QCD amplitude for $\mathcal{M}[b\bar{b}(^3P_J) \rightarrow c\bar{c}(^3S_1, P_1, \lambda_1) + c\bar{c}(^3S_1, P_2, \lambda_2)]$ and the corresponding NRQCD factorization formula.

After simple NRQCD calculation, the perturbative NRQCD matrix elements for $Q\bar{Q}(^3S_1)$ and $Q\bar{Q}(^3P_1)$ states at leading order are simply expressed as:

$$\langle Q\bar{Q}(^3S_1)|\psi^\dagger \boldsymbol{\epsilon} \cdot \boldsymbol{\sigma} \chi|0\rangle = \sqrt{2N_c}, \quad (7a)$$

$$\langle Q\bar{Q}(^3S_1)|\psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi|0\rangle = \sqrt{2N_c} \mathbf{q}^2, \quad (7b)$$

$$\frac{1}{\sqrt{3}}\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|Q\bar{Q}(^3P_0)\rangle = \sqrt{2N_c}|\mathbf{q}|, \quad (7c)$$

where $\sqrt{2N_c}$ comes from the spin and color factors of the NRQCD matrix elements and the state $|Q\bar{Q}(^{2s+1}L_J)\rangle$ is nonrelativistically normalized.

Using the formula above, we can obtain the partonic level amplitude $\mathcal{M}[b\bar{b}(^3P_0) \rightarrow c\bar{c}(^3S_1, P_1, \lambda_1) + c\bar{c}(^3S_1, P_2, \lambda_2)]$ expanded to the order v^2 :

$$\mathcal{M}_{3P_0} = 8E(q_1)E(q_2)E(Q) \sum_{m=0}^1 \sum_{n=0}^1 c_{mn}^0 \frac{1}{\sqrt{3}} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|b\bar{b}(^3P_0)\rangle \quad (8a)$$

$$\begin{aligned} & \langle c\bar{c}(^3S_1, \lambda_1)|\psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^{2m} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_1)\chi|0\rangle \langle c\bar{c}(^3S_1, \lambda_2)|\psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^{2n} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_2)\chi|0\rangle \\ & \approx (2N_c)^{3/2} 8E(q_1)E(q_2)E(Q) \left[c_{00}^0 + c_{10}^0 \frac{\mathbf{q}_1^2}{m_c^2} + c_{01}^0 \frac{\mathbf{q}_2^2}{m_c^2} + \dots \right] \end{aligned} \quad (8b)$$

In Eq. (8), we use relativistic normalization for the $c\bar{c}$ and $b\bar{b}$ states in the computation of the QCD amplitude and nonrelativistic normalization in the NRQCD matrix elements. Consequently, a factor $2E_q$ appears in the second expression of Eq. (8).

From Eq. (8), it is straightforward to extract the short-distance coefficients c_{mn}^J :

$$c_{mn}^J = \frac{m_c^{2(m+n)}}{m!n!} \frac{\partial^m}{\partial \mathbf{q}_1^{2m}} \frac{\partial^n}{\partial \mathbf{q}_2^{2n}} \left[\frac{\mathcal{M}[b\bar{b}(^3P_J) \rightarrow c\bar{c}(^3S_1, P_1, \lambda_1) + c\bar{c}(^3S_1, P_2, \lambda_2)]}{(2N_c)^{3/2} 8E(q_1)E(q_2)E(Q)} \right] \Big|_{\mathbf{q}_1^2=\mathbf{q}_2^2=0} \quad (9)$$

We can derive the LO coefficient c_{00}^J by putting $\mathbf{q} \rightarrow 0$ in the amplitude and equating E_q and m_c . While deducing the coefficient c_{01}^J, c_{10}^J , we need first expand the amplitude to the first order in \mathbf{q}^2 prior to taking the $\mathbf{q} \rightarrow 0$ limit.

III. COLOR-SINGLET MODEL CALCULATION

In this section, we present a calculation for $\chi_{bJ} \rightarrow J/\psi J/\psi$ in perturbative QCD scheme. As we discussed in Sec. II, the short-distance coefficients are insensitive to the long-distance confinement effects, and to obtain the short-distance coefficients c_{00}^J and c_{01}^J, c_{10}^J , we need only to compare the QCD amplitude and the corresponding NRQCD factorization formula in $Q\bar{Q}$ state. So we replace the physical $J/\psi, \chi_{bJ}$ states by a free $c\bar{c}$ pair of quantum number 3S_1 and a $b\bar{b}$ pair of quantum number 3P_J , respectively, and we compute the $Q\bar{Q}$ analog amplitude \mathcal{M}_{3P_J} of the hadronic amplitude $\mathcal{M}_{\chi_{bJ}}$, where the amplitude for $Q\bar{Q}$ level perturbative process $\mathcal{M}[b\bar{b}(^3P_J) \rightarrow c\bar{c}(^3S_1, P_1) + c\bar{c}(^3S_1, P_2)]$ has been aliased as \mathcal{M}_{3P_J} and the hadronic level amplitude $\mathcal{M}[\chi_{bJ} \rightarrow J/\psi + J/\psi]$ has been aliased as $\mathcal{M}_{\chi_{bJ}}$.

The Feynman diagrams for the exclusive process $\chi_{bJ}(Q, q, \lambda) \rightarrow J/\psi(P_1, q_1, \lambda_1) + J/\psi(P_2, q_2, \lambda_2)$ are shown in Fig. 1.

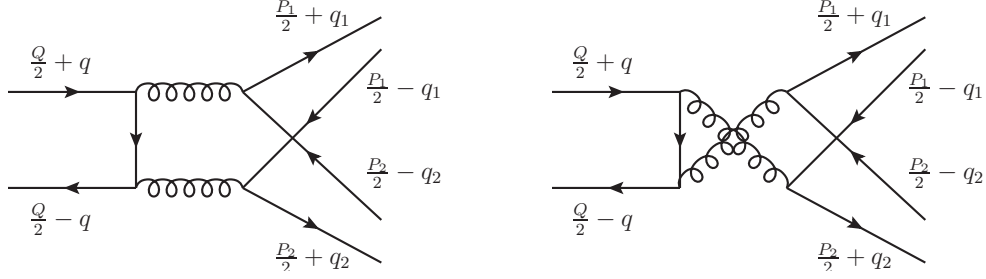


FIG. 1: Leading-order QCD Feynman diagrams that contribute to $\chi_{bJ} \rightarrow J/\psi J/\psi$

A. order- v^2 QCD amplitude

The $Q\bar{Q}$ analog $\mathcal{M}_Q(^{2s+1}L_J)$ of the hadronic can be obtained by restricting the $Q\bar{Q}$ to have an appropriate spectroscopic state. A given spin state of the color-singlet $Q\bar{Q}$ pair can be projected out by replacing $u(p)\bar{v}(\bar{p})$ or $v(\bar{p})\bar{u}(p)$ with a projection matrix that can project to a particular spin and color channel. In our case, the spins of J/ψ and χ_{bJ} are all equal to 1, so the projection matrix can be expressed as [4, 12]

$$\Pi_3^\mu \epsilon_\mu = -\frac{(\not{p} + m_Q)(\not{P} + 2E_q)\gamma^\mu(\not{p} - m_Q)}{4\sqrt{2}E_q(E_q + m_Q)}\epsilon_\mu \otimes \frac{\mathbf{1}}{\sqrt{N_c}} \quad (10a)$$

$$\gamma^0(\Pi_3^\mu \epsilon_\mu)^\dagger \gamma^0 = -\frac{(\not{p} - m_Q)\gamma^\mu(\not{P} + 2E_q)(\not{p} + m_Q)}{4\sqrt{2}E_q(E_q + m_Q)}\epsilon_\mu^* \otimes \frac{\mathbf{1}}{\sqrt{N_c}}, \quad (10b)$$

where $E_q^2 = P^2/4 = m_Q^2 - q^2$, $N_c = 3$, and $\mathbf{1}$ is the unit color matrix. ϵ is a spin polarization vector satisfying $P \cdot \epsilon = 0$ and $\epsilon \cdot \epsilon^* = -1$.

After projecting out the S -wave color-singlet spin-triplet of $c\bar{c}$ and the P -wave color-singlet spin-triplet of $b\bar{b}$, we can expand $\mathcal{M}[b\bar{b}(^3P_J) \rightarrow c\bar{c}(^3S_1, P_1, \lambda_1) + c\bar{c}(^3S_1, P_2, \lambda_2)]$ to the order of \mathbf{v}_1^2 and \mathbf{v}_2^2 , where $\mathbf{v}_i^2 = \mathbf{q}_i^2/m_c^2$. Then we can project out the diagonal, antisymmetric and symmetric traceless components of $b\bar{b}(^3P_J)$ for $J = 0, 1, 2$ as [4]. It is straightforward to obtain the $Q\bar{Q}$ analog \mathcal{M}_{3P_J} as follow:

$$\begin{aligned} \mathcal{M}_{3P_0} &= \frac{ig_s^4 2^6 (N_c^2 - 1) M_{J/\psi}^2}{3\sqrt{6} N_c^{3/2} M_{\chi_{b0}}^7} [6M_{\chi_{b0}}^2 \epsilon_1^* \cdot \epsilon_2^* - (\mathbf{v}_1^2 + \mathbf{v}_2^2) [(M_{\chi_{b0}}^2 - 2M_{J/\psi}^2) \epsilon_1^* \cdot \epsilon_2^* - 5P_1 \cdot \epsilon_2^* P_2 \cdot \epsilon_1^*]], \\ \mathcal{M}_{3P_1} &= 0, \\ \mathcal{M}_{3P_2} &= \frac{ig_s^4 2^{11/2} (1 - N_c^2) \epsilon_{\rho\sigma}}{3N_c^{3/2} M_{\chi_{b2}}^7} [3M_{\chi_{b2}}^2 (2P_1 \cdot \epsilon_2^* P_2^\rho \epsilon_1^{*\sigma} + 2P_2 \cdot \epsilon_1^* P_1^\rho \epsilon_2^{*\sigma} - 2P_1^\rho P_2^\sigma \epsilon_1^* \cdot \epsilon_2^* - M_{\chi_{b2}}^2 \epsilon_1^{*\rho} \epsilon_2^{*\sigma}) \\ &\quad + (\mathbf{v}_1^2 + \mathbf{v}_2^2) 2M_{J/\psi}^2 (2P_1^\rho P_2^\sigma \epsilon_1^* \cdot \epsilon_2^* + 2M_{\chi_{b2}}^2 \epsilon_1^{*\rho} \epsilon_2^{*\sigma} - 3\epsilon_2^{*\sigma} P_1^\rho P_2 \cdot \epsilon_1^* - 3P_1 \cdot \epsilon_2^* \epsilon_1^{*\rho} P_2^\sigma)], \end{aligned} \quad (11)$$

In Sec. II, we give the amplitude in hadronic level $\mathcal{M}[\chi_{bJ} \rightarrow J/\psi + J/\psi]$ and the $Q\bar{Q}$ analog amplitude $\mathcal{M}[b\bar{b}(^3P_J) \rightarrow c\bar{c}(^3S_1, P_1, \lambda_1) + c\bar{c}(^3S_1, P_2, \lambda_2)]$, respectively, in Eqs. (6) and (8). Then, we can derive the following relationship from the these two equations, taking χ_{b0} , for example:

$$\mathcal{M}_{\chi_{b0}} = \sqrt{\frac{2M_{J/\psi} \langle \mathcal{O}_1 \rangle_{J/\psi_1}}{2N_c (2E(q_1))^2}} \sqrt{\frac{2M_{J/\psi} \langle \mathcal{O}_1 \rangle_{J/\psi_2}}{2N_c (2E(q_2))^2}} \sqrt{\frac{2M_{\chi_{b0}} \langle \mathcal{O}_1 \rangle_{\chi_{b0}}}{2N_c (2E(Q))^2}} \mathcal{M}_{3P_0} \quad (12)$$

$$\langle \mathcal{O}_1 \rangle_{J/\psi} = |\langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \chi | 0 \rangle|^2 = \frac{N_c}{2\pi} R_{J/\psi}^2(0) \quad (13)$$

$$\langle \mathcal{O}_1 \rangle_{\chi_{b0}} = \frac{1}{3} |\langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}) | \chi_{b0} \rangle|^2 = \frac{3N_c}{2\pi} R_{\chi_{b0}}'^2(0) \quad (14)$$

As we have discussed, for the precision of our work, we have fixed $2E(q_1) = M_{J/\psi_1}$, $2E(q_2) = M_{J/\psi_2}$ and $2E(Q) = M_{\chi_{bJ}}$. Again the mass of J/ψ is identic, so we have $M_{J/\psi_1} = M_{J/\psi_2} = M_{J/\psi}$.

Substituting Eq.(11) into Eq.(12), we reach the order v^2 QCD amplitude $\mathcal{M}_{\chi_{bJ}}$:

$$\begin{aligned} \mathcal{M}_{\chi_{b0}}^s &= \frac{iA_0}{3\sqrt{6}} M_{\chi_{b0}} M_{J/\psi}^2 \left\{ -6M_{\chi_{b0}}^2 \epsilon_1^* \cdot \epsilon_2^* + (\mathbf{v}_1^2 + \mathbf{v}_2^2) [(M_{\chi_{b0}}^2 - 2M_{J/\psi}^2) \epsilon_1^* \cdot \epsilon_2^* - 5P_1 \cdot \epsilon_2^* P_2 \cdot \epsilon_1^*] \right\}, \\ \mathcal{M}_{\chi_{b1}}^s &= 0, \\ \mathcal{M}_{\chi_{b2}}^s &= \frac{iA_2\sqrt{2}}{6} M_{\chi_{b2}} \epsilon_{\rho\sigma} \left\{ 3M_{\chi_{b2}}^2 (2P_1 \cdot \epsilon_2^* P_2^\rho \epsilon_1^{*\sigma} + 2P_2 \cdot \epsilon_1^* P_1^\rho \epsilon_2^{*\sigma} - 2P_1^\rho P_2^\sigma \epsilon_1^* \cdot \epsilon_2^* - M_{\chi_{b2}}^2 \epsilon_1^{*\rho} \epsilon_2^{*\sigma}) \right. \\ &\quad \left. + (\mathbf{v}_1^2 + \mathbf{v}_2^2) 2M_{J/\psi}^2 (2P_1^\rho P_2^\sigma \epsilon_1^* \cdot \epsilon_2^* + 2M_{\chi_{b2}}^2 \epsilon_1^{*\rho} \epsilon_2^{*\sigma} - 3\epsilon_2^{*\sigma} P_1^\rho P_2 \cdot \epsilon_1^* - 3P_1 \cdot \epsilon_2^* \epsilon_1^{*\rho} P_2^\sigma) \right\}, \end{aligned} \quad (15)$$

where

$$\begin{aligned} A_0 &= \frac{-g_s^4 2^6 (N_c^2 - 1)}{N_c^3 M_{J/\psi} M_{\chi_{b0}}^{17/2}} \langle J/\psi_1 | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_1) \chi | 0 \rangle \langle J/\psi_2 | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_2) \chi | 0 \rangle \frac{1}{\sqrt{3}} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}) \psi | \chi_{b0} \rangle \quad (16) \\ A_2 &= \frac{-g_s^4 2^6 (N_c^2 - 1)}{N_c^3 M_{J/\psi} M_{\chi_{b2}}^{17/2}} \langle J/\psi_1 | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_1) \chi | 0 \rangle \langle J/\psi_2 | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda_2) \chi | 0 \rangle \sum_{ij} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}}^{(i} \sigma^{j)} \epsilon^{ij}(\lambda)) \psi | \chi_{b2} \rangle \end{aligned}$$

B. Helicity Amplitude

The polarized decay width and branching ratios can offer more useful information for both experimentation and theory, which are lost in the unpolarized ones. As we have gotten the amplitude of $\mathcal{M}[\chi_{bJ} \rightarrow J/\psi J/\psi]$, for $J = 0, 1, 2$, we can easily know the helicity amplitude by helicity amplitude formalism [26]. According to Ref. [27], we can obtain the corresponding helicity amplitude:

$$\mathcal{M}_{\lambda_1 \lambda_2; \mu}^{(J)} = \tilde{\mathcal{M}}_{\lambda_1 \lambda_2} e^{i\mu\varphi} d_{m, \lambda_1 - \lambda_2}^{(J)}(\theta), \quad (17)$$

where λ_1, λ_2 is the helicity of J/ψ , $\tilde{\mathcal{M}}_{\lambda_1 \lambda_2}$ is the reduced helicity amplitude which is a function of J, λ_1, λ_2 and particle masses but is independent of θ and φ , μ is the χ_{bJ} spin projection on fixed axe, and θ and φ are polar and azimuthal angles of one of the final J/ψ in the χ_{bJ} rest frame. We can obtained the reduced helicity amplitudes $\tilde{\mathcal{M}}$ as follows:

For χ_{b0} ,

$$\begin{aligned} \tilde{\mathcal{M}}_{0,0}^s(\chi_{b0}) &= \frac{iA_0}{12\sqrt{6}} M_{\chi_{b0}} [12M_{\chi_{b0}}^2 (2M_{J/\psi}^2 - M_{\chi_{b0}}^2) + (\mathbf{v}_1^2 + \mathbf{v}_2^2) (8M_{J/\psi}^4 - 3M_{\chi_{b0}}^4 + 12M_{J/\psi}^2 M_{\chi_{b0}}^2)] , \\ \tilde{\mathcal{M}}_{1,1}^s(\chi_{b0}) &= \frac{iA_0}{3\sqrt{6}} M_{\chi_{b0}} M_{J/\psi}^2 [6M_{\chi_{b0}}^2 + (\mathbf{v}_1^2 + \mathbf{v}_2^2) (2M_{J/\psi}^2 - M_{\chi_{b0}}^2)] , \end{aligned} \quad (18)$$

For χ_{b2} ,

$$\begin{aligned}
\tilde{\mathcal{M}}_{0,0}^s(\chi_{b2}) &= \frac{iA_2}{6\sqrt{3}} M_{\chi_{b2}} [3M_{\chi_{b2}}^2 (4M_{J/\psi}^2 + M_{\chi_{b2}}^2) - (\mathbf{v}_1^2 + \mathbf{v}_2^2) 2M_{J/\psi}^2 (3M_{\chi_{b2}}^2 + 4M_{J/\psi}^2)] , \\
\tilde{\mathcal{M}}_{1,1}^s(\chi_{b2}) &= \frac{iA_2}{3\sqrt{3}} M_{\chi_{b2}} [6M_{\chi_{b2}}^2 M_{J/\psi}^2 - (\mathbf{v}_1^2 + \mathbf{v}_2^2) M_{J/\psi}^2 (M_{\chi_{b2}}^2 + 4M_{J/\psi}^2)] , \\
\tilde{\mathcal{M}}_{1,0}^s(\chi_{b2}) &= \frac{iA_2}{12} M_{J/\psi} M_{\chi_{b2}}^2 [12M_{\chi_{b2}}^2 - (\mathbf{v}_1^2 + \mathbf{v}_2^2) (M_{\chi_{b2}}^2 + 12M_{J/\psi}^2)] , \\
\tilde{\mathcal{M}}_{1,-1}^s(\chi_{b2}) &= \frac{iA_2\sqrt{2}}{6} M_{\chi_{b2}}^3 [3M_{\chi_{b2}}^2 - 4(\mathbf{v}_1^2 + \mathbf{v}_2^2) M_{J/\psi}^2] , \tag{19}
\end{aligned}$$

According to parity invariance, there are only two independent helicity amplitudes for χ_{b0} and four independent helicity amplitudes for χ_{b2} . The unpolarized amplitude squared can be obtained by integrating the polar angle and summing all the helicity states. The relationship between the unpolarized amplitude squared and the helicity amplitude squared is:

$$|\mathcal{M}_{\chi_{b0}}^s|^2 = |\tilde{\mathcal{M}}_{0,0}^s(\chi_{b0})|^2 + 2|\tilde{\mathcal{M}}_{1,1}^s(\chi_{b0})|^2 \tag{20a}$$

$$|\mathcal{M}_{\chi_{b2}}^s|^2 = |\tilde{\mathcal{M}}_{0,0}^s(\chi_{b2})|^2 + 2|\tilde{\mathcal{M}}_{1,1}^s(\chi_{b2})|^2 + 4|\tilde{\mathcal{M}}_{1,0}^s(\chi_{b2})|^2 + 2|\tilde{\mathcal{M}}_{1,-1}^s(\chi_{b2})|^2 \tag{20b}$$

Here we give some general prediction of these helicity amplitudes by hadron helicity selection rule. In this exclusive decay process, when we consider the lowest-order strong interaction in the limit $m_b \rightarrow \infty$ with m_c fixed, we can obtain the asymptotic behavior for $\text{Br}[\chi_{bJ} \rightarrow J/\psi J/\psi]$:

$$\text{Br}_{\text{str}}[\chi_{bJ} \rightarrow J/\psi(\lambda_1) + J/\psi(\lambda_2)] \sim \alpha_s^2 v_c^6 \left(\frac{m_c^2}{m_b^2} \right)^{2+|\lambda_1+\lambda_2|} , \tag{21}$$

where v_c is the relative velocity of $c\bar{c}$ in J/ψ , and the factor v^6 comes from the wave function at the origin of J/ψ .

From Eq. (21), we can see clearly that the helicity configurations of two J/ψ decide the scaling behavior of the branching ratio. We figure out that when the decay configuration is $\lambda_1 + \lambda_2 = 0$, i.e. $(\lambda_1, \lambda_2) = (0, 0)$, and $(\lambda_1, \lambda_2) = (1, -1)$, the branching ration exhibits the slowest asymptotic decrease $\text{Br}_{\text{str}} \sim 1/m_b^4$. And we can also expect $\text{Br}_{\text{str}} \sim 1/m_b^6$ when $(\lambda_1, \lambda_2) = (0, 1)$ and the $\text{Br}_{\text{str}} \sim 1/m_b^8$ when $(\lambda_1, \lambda_2) = (1, 1)$. In fact, due to the nonzero charm mass helicity conservation is violated.

In particular, the helicity state $(0, 0)$ of the channel $\chi_{b1} \rightarrow J/\psi J/\psi$ is zero because it is strictly forbidden due to the conflict between parity and angular momentum conservation. We call this kind of process an unnatural decay process [28].

C. Electromagnetic amplitude

For completeness, we also consider the electromagnetic contributions to $\chi_{bJ} \rightarrow J/\psi J/\psi$. There are four QED diagrams, two of which have the same topology as Fig.1, but with gluons replaced by photons, lead to the amplitude that has the same form as Eq. (16) except α_s^2 is replaced by $e_b^2 e_c^2 \alpha^2$. But their contributions are much more suppressed than those from the fragmentation diagrams in Fig.2, and we will not consider them.

We can see that in Fig.2, both J/ψ decay from the corresponding virtual photon, so we can use their decay constant instead of the vacuum matrix element to describe them. With the definition given in Ref. [29],

$$\langle 0 | \bar{c} \gamma^\mu c | J/\psi(P, \epsilon) \rangle = f_{J/\psi} M_{J/\psi} \epsilon^\mu \quad (22)$$

where P is the momentum of J/ψ , ϵ is its polarization vector, and $f_{J/\psi}$ is the so-called decay constant.

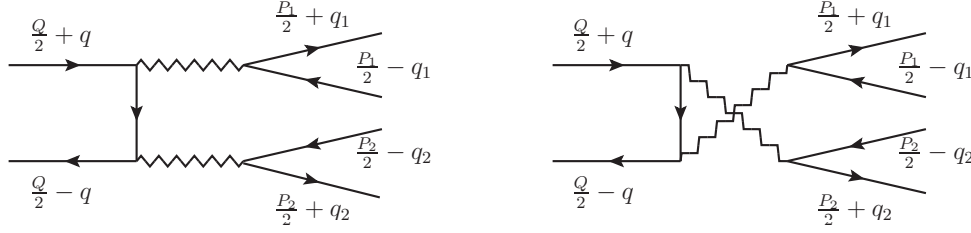


FIG. 2: Lowest-order QED diagrams that contribute to $\chi_{bJ} \rightarrow J/\psi + J/\psi$. Only the fragmentation-type diagrams are retained, whereas the other two, which can be obtained by replacing the gluons in Fig.1 with photons, have been suppressed.

We can infer the QED fragmentation contribution to the amplitude in Fig. 2; in the following results, we have already included all order relativistic corrections by using the decay constant:

$$\mathcal{M}_{\chi_{b0}}^{em} = \frac{i2B_0}{\sqrt{6}} \frac{M_{J/\psi}^2}{M_{\chi_{b0}}} [(3M_{\chi_{b0}}^2 - 8M_{J/\psi}^2) \epsilon_1^* \cdot \epsilon_2^* M_{\chi_{b0}}^2 + 2(2M_{J/\psi}^2 - 3M_{\chi_{b0}}^2) P_1 \cdot \epsilon_2^* P_2 \cdot \epsilon_1^*] \quad (23a)$$

$$\mathcal{M}_{\chi_{b1}}^{em} = 8B_1 \frac{M_{J/\psi}^4}{M_{\chi_{b1}}^2} \epsilon_{\alpha\beta\rho\sigma} Q^\alpha \epsilon^\beta (P_1^\sigma \epsilon_1^{*\rho} P_1 \cdot \epsilon_2^* + P_1^\rho \epsilon_2^{*\sigma} P_2 \cdot \epsilon_1), \quad (23b)$$

$$\mathcal{M}_{\chi_{b2}}^{em} = i2B_2 \sqrt{2} M_{J/\psi}^2 M_{\chi_{b2}} \epsilon_{\rho\sigma} [2\epsilon_1^{*\rho} P_2^\sigma P_1 \cdot \epsilon_2^* + 2\epsilon_2^{*\rho} P_1^\sigma P_2 \cdot \epsilon_1^* + (2M_{J/\psi}^2 - m_b^2) \epsilon_2^{*\rho} \epsilon_1^{*\sigma} + 2P_1^\rho P_1^\sigma \epsilon_1^* \cdot \epsilon_2^*], \quad (23c)$$

where

$$B_0 = -e_b^2 e_c^2 e^4 f_{J/\psi}^2 \frac{4(M_{\chi_{b0}}^2 - 2M_{J/\psi}^2)^{-2}}{\sqrt{M_{\chi_{bJ}}} M_{J/\psi}^4} \frac{1}{\sqrt{3}} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}) \psi | \chi_{b0} \rangle, \quad (24a)$$

$$B_1 = -e_b^2 e_c^2 e^4 f_{J/\psi}^2 \frac{4(M_{\chi_{b1}}^2 - 2M_{J/\psi}^2)^{-2}}{\sqrt{M_{\chi_{bJ}}} M_{J/\psi}^4} \frac{1}{\sqrt{2}} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda)) \psi | \chi_{b1} \rangle, \quad (24b)$$

$$B_2 = -e_b^2 e_c^2 e^4 f_{J/\psi}^2 \frac{4(M_{\chi_{b2}}^2 - 2M_{J/\psi}^2)^{-2}}{\sqrt{M_{\chi_{bJ}}} M_{J/\psi}^4} \sum_{ij} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{D}^{(i} \sigma^{j)} \epsilon^{ij}(\lambda)) \psi | \chi_{b2} \rangle. \quad (24c)$$

Following are the helicity amplitudes:

$$\tilde{\mathcal{M}}_{0,0}^{em}(\chi_{b0}) = iB_0 \frac{16}{\sqrt{6}} M_{\chi_{b0}} M_{J/\psi}^4, \tilde{\mathcal{M}}_{1,1}^{em}(\chi_{b0}) = iB_0 \frac{4}{\sqrt{6}} M_{\chi_{b0}} M_{J/\psi}^2 (8M_{J/\psi}^2 - 3M_{\chi_{b0}}^2), \quad (25a)$$

$$\tilde{\mathcal{M}}_{1,0}^{em}(\chi_{b1}) = iB_1 4M_{J/\psi}^3 (4M_{J/\psi}^2 - M_{\chi_{b1}}^2), \quad (25b)$$

$$\tilde{\mathcal{M}}_{0,0}^{em}(\chi_{b2}) = iB_2 \frac{16}{\sqrt{3}} M_{J/\psi}^4 M_{\chi_{b2}}, \tilde{\mathcal{M}}_{1,1}^{em}(\chi_{b2}) = iB_2 \frac{8}{\sqrt{3}} M_{J/\psi}^4 M_{\chi_{b2}}, \quad (25c)$$

$$\tilde{\mathcal{M}}_{1,0}^{em}(\chi_{b2}) = iB_2 4M_{\chi_{b2}}^2 M_{J/\psi}^3, \quad \tilde{\mathcal{M}}_{1,-1}^{em}(\chi_{b2}) = iB_2 4\sqrt{2} M_{J/\psi}^2 M_{\chi_{b2}} (M_{\chi_{b2}}^2 - 2M_{J/\psi}^2),$$

The relationship between the unpolarized amplitude squared and the helicity amplitude squared is similar to the relation in the QCD section except a new equation for χ_{b1} :

$$|\mathcal{M}_{\chi_{b1}}^{em}|^2 = 4|\tilde{\mathcal{M}}_{1,0}^{em}(\chi_{b1})|^2 \quad (26)$$

D. Decay width

As we have known the amplitude squared, we can easily derive the decay width of $\chi_{bJ} \rightarrow J/\psi J/\psi$. The polarized decay width can be expressed as

$$\Gamma_{\lambda_1, \lambda_2} = \frac{1}{2M_{\chi_{bJ}}} \times \frac{1}{2} \times \Phi_2 \int_{-1}^1 \frac{d \cos \theta}{2} |\mathcal{M}_{\lambda_1 \lambda_2; \mu}^{(J)}|^2 \quad (27)$$

The unpolarized decay width can be expressed as

$$\Gamma_{unp} = \frac{1}{2M_{\chi_{bJ}}} \times \frac{1}{2} \times \Phi_2 \int_{-1}^1 \frac{d \cos \theta}{2} \sum_{\lambda_1 \lambda_2} |\mathcal{M}_{\lambda_1 \lambda_2; \mu}^{(J)}|^2 \quad (28)$$

where $|\mathcal{M}_{\lambda_1 \lambda_2; \mu}^{(J)}|^2 = |\mathcal{M}_{\lambda_1 \lambda_2; \mu s}^{(J)} - \mathcal{M}_{\lambda_1 \lambda_2; \mu em}^{(J)}|^2$, $\Phi_2 = \frac{1}{8\pi} \sqrt{1 - \frac{4M_{J/\psi}^2}{M_{\chi_{bJ}}^2}}$.

IV. NUMERICAL ANALYSIS

A. Input parameters

In this section, we will apply the results obtained in Sec. III to give some phenomenological predictions. Before we carry out the numerical calculations, we need to fix several input parameters, such as $M_{\chi_{bJ}}$, $M_{J/\psi}$, α , α_s , $\langle \mathcal{O}_1 \rangle_{J/\psi}$, $\langle \mathcal{O}_1 \rangle_{\chi_{bJ}}$, $f_{J/\psi}$ and $\langle v^2 \rangle_{J/\psi}$.

First, we need to fix the values of coupling constants; in our work, we set the running QCD strong coupling constant $\alpha_s(M_{\chi_{bJ}}/2) = 0.22$ by using the two-loop formula with $\Lambda_{\overline{\text{MS}}} = 0.338$ GeV [8, 9] and the electromagnetic fine structure constant $\alpha = 1/137$.

Next, The NRQCD matrix element $\langle \mathcal{O}_1 \rangle_{\chi_{bJ}}$ can be obtained from the derivative of radial wave functions near the origin in the potential model, for P-wave is $\langle \mathcal{O}_1 \rangle_{\chi_{bJ}} \approx \frac{3N_c}{2\pi} |R'_{1P}(0)|^2$ as in Ref.[4, 30]. In Ref.[31] the values of $|R'_{1P}(0)|^2$ for four potentials have been listed; we use the value of the Cornell potential, and then obtain $\langle \mathcal{O}_1 \rangle_{\chi_{bJ}} = 2.96 \text{ GeV}^5$. The color-singlet NRQCD matrix elements of J/ψ , i.e. $\langle \mathcal{O}_1 \rangle_{J/\psi}$, can be obtained from the electromagnetic

decay rate of the J/ψ in which the α_s leading order is considered. Using the measured dielectric width 5.5 KeV, we obtained $\langle \mathcal{O}_1 \rangle_{J/\psi} = 0.268 \text{ GeV}^3$.

The values for the physical masses of the involving hadrons are taken from Ref.[32] as $M_{\chi_{b0}} = 9.859 \text{ GeV}$, $M_{\chi_{b1}} = 9.893 \text{ GeV}$, $M_{\chi_{b2}} = 9.912 \text{ GeV}$, $M_{J/\psi} = 3.097 \text{ GeV}$. The decay constant for J/ψ is taken as $f_{J/\psi} = 0.406 \text{ GeV}$ as in Ref. [29].

Considering the order v^2 relativistic corrections are calculated, we also need to know the value of $\langle v^2 \rangle_{J/\psi}$. Here we adopt the value $\langle v^2 \rangle_{J/\psi_1} = \langle v^2 \rangle_{J/\psi_2} = 0.225$ extracted from the recent Cornell-potential-model-based analysis in Ref.[33].

B. The decay width of $\chi_{bJ} \rightarrow J/\psi J/\psi$

With the input parameters fixed, we present the numerical results of the polarized and unpolarized decay widths for $\chi_{bJ} \rightarrow J/\psi J/\psi$ in Table I.

TABLE I: The polarized and unpolarized decay widths for $\chi_{bJ} \rightarrow J/\psi J/\psi$, where we list the leading-order QCD contributions of $\Gamma(\chi_{bJ} \rightarrow J/\psi J/\psi)$, QED corrections, v^2 -order QCD corrections (including LO contributions) and both the QED and QCD corrections. The values are all in units of eV.

		$\Gamma_{0,0}$	$\Gamma_{1,1}$	$\Gamma_{1,0}$	$\Gamma_{1,-1}$	Γ_{unp}
LO	χ_{b0}	4.334	0.262	—	—	4.859
	χ_{b1}	—	—	—	—	—
	χ_{b2}	1.251	0.099	0.758	3.881	12.240
QED	χ_{b0}	4.338	0.266	—	—	4.870
	χ_{b1}	—	—	2.26×10^{-7}	—	9.04×10^{-7}
	χ_{b2}	1.250	0.098	0.757	3.872	12.216
v^2 -QCD	χ_{b0}	5.067	0.231	—	—	5.530
	χ_{b1}	—	—	—	—	—
	χ_{b2}	1.078	0.079	0.639	3.439	10.673
QED&QCD	χ_{b0}	5.070	0.235	—	—	5.541
	χ_{b1}	—	—	2.26×10^{-7}	—	9.04×10^{-7}
	χ_{b2}	1.077	0.079	0.638	3.431	10.651

We can see that the relative order- v^2 contributions are very prominent. It is found to increase the lower-order prediction for the decay width by about 13.8% for $\chi_{b0} \rightarrow J/\psi J/\psi$, and about 12.8% for $\chi_{b2} \rightarrow J/\psi J/\psi$. The QED contributions are very small, only about 0.2% for $\chi_{b0} \rightarrow J/\psi J/\psi$ and $\chi_{b2} \rightarrow J/\psi J/\psi$.

We can also see in Table I that the polarized decay width is consistent with the helicity selection rule in general. As we analyzed in Sec. III, the configurations of two J/ψ with $(\lambda_1, \lambda_2)=(0, 0)$ and $(\lambda_1, \lambda_2)=(1, -1)$ bear the smallest suppression and contribute the largest component to total decay width. The $(\lambda_1, \lambda_2)=(0, 1)$ channel gives the second largest contribution and the contribution from the $(\lambda_1, \lambda_2)=(1, 1)$ channel is the smallest one. So we speculate the most observable channels are $(\lambda_1, \lambda_2)=(1, -1)$ for χ_{b2} and $(\lambda_1, \lambda_2)=(0, 0)$ for χ_{b0} respectively.

C. The decay branching ratio of $\chi_{bJ} \rightarrow J/\psi J/\psi$

Nevertheless, what we are more interested in is the branching ratio, so we need to know the total decay width. According to Ref. [7], the decay rate of χ_{bJ} into light hadrons (LH) is

$$\Gamma(\chi_{bJ} \rightarrow \text{LH}) = \frac{2\text{Im} f_1(^3P_J)}{m_b^4} \langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}} + \frac{2\text{Im} f_8(^3S_1)}{m_b^2} \langle \mathcal{O}_8(^1S_0) \rangle_{\chi_{bJ}}. \quad (29)$$

The imaginary parts of $f_1(^3P_J)$ and $f_8(^1S_0)$ have been calculated up to order α_s^3 in Ref. [34], but for $f_1(^3P_0)$ and $f_1(^3P_2)$ different results were given in Ref.[35], so we take the leading-order results as

$$\Gamma_{tot}(\chi_{b0}) = \frac{64\pi\alpha_s^2 \langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}}}{3M_{\chi_{b0}}^4} \left(1 + \frac{n_f M_{\chi_{b0}}^2 \langle \mathcal{O}_8(^1S_0) \rangle_{\chi_{bJ}}}{16 \langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}}} \right), \quad (30a)$$

$$\Gamma_{tot}(\chi_{b1}) = \frac{64\pi\alpha_s^2 \langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}}}{3M_{\chi_{b1}}^4} \left(0 + \frac{n_f M_{\chi_{b1}}^2 \langle \mathcal{O}_8(^1S_0) \rangle_{\chi_{bJ}}}{16 \langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}}} \right), \quad (30b)$$

$$\Gamma_{tot}(\chi_{b2}) = \frac{64\pi\alpha_s^2 \langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}}}{3M_{\chi_{b2}}^4} \left(\frac{4}{15} + \frac{n_f M_{\chi_{b2}}^2 \langle \mathcal{O}_8(^1S_0) \rangle_{\chi_{bJ}}}{16 \langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}}} \right), \quad (30c)$$

where $n_f = 4$. According to Ref. [30], we take $\frac{\langle \mathcal{O}_8(^1S_0) \rangle_{\chi_{bJ}}}{\langle \mathcal{O}_1(^3P_J) \rangle_{\chi_{bJ}}} = 0.0021 \text{ GeV}^{-2}$, then we can obtain the value of total decay.

Another method is to calculate the decay width of E1 transitions of $\chi_{bJ} \rightarrow \gamma \Upsilon$, according to the branching ratio of $\chi_{bJ} \rightarrow \gamma \Upsilon$ from Ref.[36], we can obtain the total decay width of χ_{bJ} . The E1 transitions width is defined as [37]

$$\Gamma(1^3P_J \xrightarrow{E1} 1^3S_1 + \gamma) = \frac{4\alpha e_Q^2}{3} \frac{1}{3} k^3 \left| \frac{3}{k} \int_0^\infty r^2 dr R_{11}(r) R_{10}(r) \left[\frac{kr}{2} j_0\left(\frac{kr}{2}\right) - j_1\left(\frac{kr}{2}\right) \right] \right|^2 \quad (31)$$

where k is the photon momentum, and $R_{nl}(r)$ is radial wave-function.

The total decay widths of χ_{bJ} obtained from the two methods are listed in Table II.

TABLE II: Total decay width of χ_{bJ} . One is obtained by E1 transitions, which is from Ref. [37] and PDG data. Another is from the χ_{bJ} decay into LH in leading order. The values are all in units of KeV.

	E1	LH
$\Gamma(\chi_{b0})$	> 319	1068
$\Gamma(\chi_{b1})$	69	52
$\Gamma(\chi_{b2})$	124	317

From all these values we can obtain the branching ratio in Table III. For comparison, we juxtapose the decay width of $\chi_{bJ} \rightarrow J/\psi J/\psi$ from our calculation and the results obtained in [23] within NRQCD, where the first errors come from the uncertainties in the distribution for final-state mesons, the second in α_s , and the last errors are associated with power-law corrections.

TABLE III: Total decay width and branching ratio of $\chi_{bJ} \rightarrow J/\psi J/\psi$. Here we give the total decay width including QED and order- v^2 relativistic corrections. For comparison, we list the results obtained in [23] within NRQCD. We also show the branching ratios obtained by χ_{bJ} decay into light hadrons and E1 transition.

[h]	$\Gamma(\text{eV})$	$\Gamma[23](\text{eV})$	$\text{Br}_{\text{E1}}(10^{-5})$	$\text{Br}_{\text{LH}}(10^{-5})$
$\chi_{b0} \rightarrow J/\psi J/\psi$	5.54	$27_{-2.5}^{+5} \pm 19 \pm 13$	< 1.7	0.5
$\chi_{b1} \rightarrow J/\psi J/\psi$	9.04×10^{-7}	—	1.4×10^{-6}	1.9×10^{-6}
$\chi_{b2} \rightarrow J/\psi J/\psi$	10.6	$65_{-12}^{+14} \pm 46 \pm 32$	8.6	3.4

We can see that the results obtained in [23] are in general larger than ours. We have compared our analytical expressions with [23] and found a difference by a factor of 2 in the formula of decay rate which will reduce the result of [23] half. In addition, the various input parameters such as NRQCD matrix elements, α_s , and particle masses can bring the large uncertainties. Finally, because we expand the pole mass m_c around the physical mass $M_{J/\psi}$ instead of the traditional matching method, this will decrease our predictions comparing to the orthodox method. Although the results seem to have a sensible difference, we believe the order of magnitude is correct.

D. Some predictions of observation potential for $\chi_{bJ} \rightarrow J/\psi J/\psi$

We use the branching ratio given in the Table III to explore the possibility for these processes to be observed in experiments. Considering the branching ratio for each of the decays $J/\psi \rightarrow \mu^+ \mu^-$ is about 6%, we find the branching ratio of χ_{bJ} decay to muon pair is $\text{Br}[\chi_{b0} \rightarrow J/\psi J/\psi \rightarrow 4\mu] \approx 0.6 \times 10^{-7}$ and $\text{Br}[\chi_{b2} \rightarrow J/\psi J/\psi \rightarrow 4\mu] \approx 3.1 \times 10^{-7}$. The total cross section for χ_{b0} and χ_{b2} production at Tevatron energy according to [14] is

$$\begin{aligned}\sigma(p\bar{p} \rightarrow \chi_{b0} + X) &= 250 \text{ nb}, \\ \sigma(p\bar{p} \rightarrow \chi_{b2} + X) &= 320 \text{ nb},\end{aligned}\tag{32}$$

We estimate there are about $50 \sim 150$ for χ_{b0} and $400 \sim 1000$ for χ_{b2} produced events in Tevatron Run 2 that achieve an integrated luminosity of about 10 fb^{-1} by April 2011.

Similarly, we can combine the cross sections at LHC, which are about 6 times larger than the corresponding cross sections at Tevatron [14], to predict the number of produced events that may reach between 1500 and 4500 for χ_{b0} and between 12000 and 30000 for χ_{b2} with the accumulated luminosity 50 fb^{-1} of LHC in 2010, and with the acceptance and efficiency of detector considered, we expect $15 \sim 45$ and $120 \sim 300$ observed events per year for χ_{b0} and χ_{b2} respectively.

E. The decay width and branching ratio of $\chi_{cJ} \rightarrow VV (V \rightarrow \omega, \phi)$

The constituent quark model, which treats the light mesons as non-relativistic bound states, is also frequently invoked as an alternative method for a quick order-of-magnitude estimate. In this sense, the preceding formulas derived for $\chi_{bJ} \rightarrow J/\psi J/\psi$ can be applied to describe the decay processes $\chi_{cJ} \rightarrow VV$, once we understand that we are working with the constituent quark model.

TABLE IV: Parameters used for numerical calculation in $\chi_{cJ} \rightarrow \phi\phi$ and $\chi_{cJ} \rightarrow \omega\omega$

	$M_V(\text{GeV})$	α_s	$ R_{sV}(0) ^2(\text{GeV}^3)$	$ R'_{\chi_c} ^2(\text{GeV}^5)$	$M_{\chi_{c0}}(\text{GeV})$	$M_{\chi_{c1}}(\text{GeV})$	$M_{\chi_{c2}}(\text{GeV})$
$\chi_{cJ} \rightarrow \omega\omega$	0.78265	0.3	0.11	0.075	3.41475	3.51066	3.55620
$\chi_{cJ} \rightarrow \phi\phi$	1.019455	0.3	0.19	0.075	3.41475	3.51066	3.55620

TABLE V: Decay widths and Branching ratios of $\chi_{cJ} \rightarrow \phi\phi$ and $\chi_{cJ} \rightarrow \omega\omega$. The second and third rows are the results of our calculation. The next two rows are the results obtained in [14] within “ ϕ_3 ” model. The next row is the branching fractions obtained by BESIII [39], where we take the results of combined final state. The last row is the PDG [32] results. In $\chi_{cJ} \rightarrow \omega\omega$, there are only our results, BESIII results and PDG results.

	$\chi_{c0} \rightarrow \phi\phi$	$\chi_{c1} \rightarrow \phi\phi$	$\chi_{c2} \rightarrow \phi\phi$
$\Gamma(\text{keV})$	3.3	1.9×10^{-7}	5.9
$\text{Br}(10^{-4})$	3.2	2.2×10^{-6}	30
$\Gamma(\text{keV})$ [14]	2.10	—	3.38
$\text{Br}(10^{-4})$ [14]	3.01	—	21.3
$\text{Br}(10^{-4})$ [39](BESIII)	$8.0 \pm 0.3 \pm 0.8$	$4.4 \pm 0.2 \pm 0.5$	$10.7 \pm 0.3 \pm 1.2$
$\text{Br}(10^{-4})$ [32](PDG)	9.2 ± 1.9	—	14.8 ± 2.8
	$\chi_{c0} \rightarrow \omega\omega$	$\chi_{c1} \rightarrow \omega\omega$	$\chi_{c2} \rightarrow \omega\omega$
$\Gamma(\text{keV})$	2.3	2.2×10^{-7}	3.2
$\text{Br}(10^{-4})$	2.2	2.6×10^{-6}	16
$\text{Br}(10^{-4})$ [39](BESIII)	$9.5 \pm 0.3 \pm 1.1$	$6.0 \pm 0.2 \pm 0.7$	$8.9 \pm 0.3 \pm 1.1$
$\text{Br}(10^{-4})$ [32](PDG)	22 ± 7	—	19.0 ± 6.0

We have listed the numerical values for parameters in Table. IV.

We take $\chi_{cJ} \rightarrow \phi\phi$ as an representative. By regarding ϕ as a strangeonium, we can directly use Eq. (27), only with some trivial changes of input parameters. We take the constituent quark mass $m_s \approx M_\phi/2 = 0.5 \text{ GeV}$. The radial wave function at the origin of ϕ , $|R_\phi(0)|^2$, can be extracted analogously from its measured dielectron width of 0.19 GeV^3 . Taking $M_{\chi_{c0}} = 3.41475 \text{ GeV}$, and the strong coupling constant $\alpha_s = 0.3$, and only considering the contribution of the leading-order QCD and QED contributions, we can obtain the decay width and branching ration of $\chi_{cJ} \rightarrow \phi\phi$ in Table V.

It is also interesting to consider the similar decay process $\chi_{cJ} \rightarrow \omega\omega$, Parallel to the preceding procedure, we also give the results in Table V.

From the Table V, we can see our predictions of $\text{Br}[\chi_{c0} \rightarrow \phi\phi]$ and $\text{Br}[\chi_{c2} \rightarrow \phi\phi]$ are 3.2×10^{-4} and 30×10^{-4} respectively. Our results are compatible with the results obtained in [14] within “ ϕ_3 ” model [38] not only in decay width but also in branching ratio. Moreover, the branching ratio of $\chi_{c0,2} \rightarrow \phi\phi$ is also compatible with BESIII [39] measurement and PDG [32] in the order of magnitude. For the process $\chi_{cJ} \rightarrow \omega\omega$, We can learn from Table V that our result of $\text{Br}[\chi_{c0} \rightarrow \omega\omega]$ is close to the observation in BESIII [39], while for $\text{Br}[\chi_{c2} \rightarrow \omega\omega]$, our result is close to the PDG [32]. It is noting that both of the observation in BESIII [39] and the result in PDG [32], the branching ratio of $\chi_{c0} \rightarrow \omega\omega$ is large than the one of $\chi_{c2} \rightarrow \omega\omega$, while our prediction is opposite. This discrepancy may be caused by the

model approximation.

V. CONCLUSION

The major task of this work is to calculate the first-order relativistic corrections and electromagnetic corrections of $\chi_{bJ} \rightarrow J/\psi J/\psi$ in the context of NRQCD factorization. We first introduced NRQCD factorization formula, which is particularly suitable for calculating the relativistic corrections to quarkonium production and decay processes in the color-singlet channel. Then we compared the orthodox matching strategy with our matching method and emphasized that our approach avoids the complications that come from squared amplitude and phase-space integral. These two methods are equivalent thanks to the Gremm-Kapustin relation and we give our matching schema at amplitude level.

As a phenomenological application of our results, we calculate the decay width and branching ratio of $\chi_{bJ} \rightarrow J/\psi J/\psi$. The conclusion is that the relativistic contributions of $\chi_{bJ} \rightarrow J/\psi J/\psi$ are modest, about 13.8% for $\chi_{b0} \rightarrow J/\psi J/\psi$, and about 12.8% for $\chi_{b2} \rightarrow J/\psi J/\psi$. The QED contributions are very small, only about 0.2% for $\chi_{b0} \rightarrow J/\psi J/\psi$ and $\chi_{b2} \rightarrow J/\psi J/\psi$. The more interesting branching ratios are predicted to be of order 10^{-5} for $\chi_{b0} \rightarrow J/\psi J/\psi$, $\chi_{b2} \rightarrow J/\psi J/\psi$, but 10^{-11} for $\chi_{b1} \rightarrow J/\psi J/\psi$, since there is only electromagnetism contribution in this channel. With the result, we predict it is possible to observe these processes in LHC.

Finally, as an exploratory work, we also have a quick order-of-magnitude estimate of the decay width and branching ratio of $\chi_{cJ} \rightarrow \omega\omega, \phi\phi$ by our formula in leading-order QCD and QED contribution working with the constituent quark model. The predict of $\text{Br}[\chi_{c0,2} \rightarrow \phi\phi]$ is consistent with the experimental result in the order of magnitude.

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- [1] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **89**, 142001 (2002), [arXiv:hep-ex/0205104].
 - [2] K. Abe *et al.* [Belle Collaboration], Phys. Rev. D **70**, 071102 (2004), hep-ex/0407009.
 - [3] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. **D72**, 031101 (2005). [hep-ex/0506062].
 - [4] E. Braaten and J. Lee, Phys. Rev. D **67**, 054007 (2003) [arXiv:hep-ph/0211085];
 - [5] K. Y. Liu, Z. G. He and K. T. Chao, Phys. Lett. B **557**, 45 (2003) [arXiv:hep-ph/0211181].
 - [6] K. -Y. Liu, Z. -G. He, K. -T. Chao, Phys. Rev. **D77**, 014002 (2008). [hep-ph/0408141].
 - [7] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995) [Erratum-ibid. D **55**, 5853 (1997)] [arXiv:hep-ph/9407339].
 - [8] Y. -J. Zhang, Y. -j. Gao, K. -T. Chao, Phys. Rev. Lett. **96**, 092001 (2006). [hep-ph/0506076].

- [9] B. Gong, J. -X. Wang, Phys. Rev. **D77**, 054028 (2008). [arXiv:0712.4220 [hep-ph]].
- [10] K. Hagiwara, E. Kou and C. F. Qiao, Phys. Lett. B **570** (2003) 39 [arXiv:hep-ph/0305102].
- [11] Z. G. He, Y. Fan and K. T. Chao, Phys. Rev. D **75**, 074011 (2007) [arXiv:hep-ph/0702239].
- [12] G. T. Bodwin, J. Lee and C. Yu, Phys. Rev. D **77**, 094018 (2008) [arXiv:0710.0995 [hep-ph]].
- [13] V. V. Braguta, Phys. Rev. **D79**, 074018 (2009). [arXiv:0811.2640 [hep-ph]].
- [14] V. V. Braguta, A. K. Likhoded and A. V. Luchinsky, Phys. Rev. D **72**, 094018 (2005) [arXiv:hep-ph/0506009].
- [15] G. Hao, Y. Jia, C. F. Qiao and P. Sun, JHEP **0702** (2007) 057 [arXiv:hep-ph/0612173].
- [16] Y. Jia, Phys. Rev. D **76** (2007) 074007 [arXiv:0706.3685 [hep-ph]].
- [17] E. Braaten, S. Fleming and A. K. Leibovich, Phys. Rev. D **63**, 094006 (2001) [arXiv:hep-ph/0008091].
- [18] F. Maltoni and A. D. Polosa, Phys. Rev. D **70** (2004) 054014 [arXiv:hep-ph/0405082].
- [19] Y. Jia, Phys. Rev. D **78**, 054003 (2008) [arXiv:hep-ph/0611130].
- [20] B. Gong, Y. Jia and J. X. Wang, Phys. Lett. B **670** (2009) 350 [arXiv:0808.1034 [hep-ph]].
- [21] V. V. Braguta and V. G. Kartvelishvili, Phys. Rev. D **81** (2010) 014012 [arXiv:0907.2772 [hep-ph]].
- [22] P. Sun, G. Hao and C. F. Qiao, Phys. Lett. B **702**, 49 (2011) [arXiv:1005.5535 [hep-ph]].
- [23] V. V. Braguta, A. K. Likhoded and A. V. Luchinsky, Phys. Atom. Nucl. **73** (2010) 1054 [Yad. Fiz. **73** (2010) 1091].
- [24] J. Zhang, *Exclusive decay of P-wave Bottomonium into double J/ψ* , MS thesis, IHEP, CAS and ShangXi University (2009).
- [25] M. Gremm and A. Kapustin, Phys. Lett. B **407**, 323 (1997) [arXiv:hep-ph/9701353].
- [26] M. Jacob and G. C. Wick, Annals Phys. **7**, 404 (1959) [Annals Phys. **281**, 774 (2000)].
- [27] H. E. Haber, arXiv:hep-ph/9405376.
- [28] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. **112**, 173 (1984);
- [29] D. S. Hwang, G. -H. Kim, Z. Phys. **C76**, 107-110 (1997). [hep-ph/9703364].
- [30] G. T. Bodwin, E. Braaten, D. Kang and J. Lee, Phys. Rev. D **76**, 054001 (2007) [arXiv:0704.2599 [hep-ph]].
- [31] E. J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995) [arXiv:hep-ph/9503356].
- [32] K. Nakamura, *et al.* (Particle Data Group) JP ,G **37**, 075021 (2010)
- [33] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **101**, 071801 (2008). [arXiv:0807.1086 [hep-ex]].
- [34] A. Petrelli, M. Cacciari, M. Greco, F. Maltoni and M. L. Mangano, Nucl. Phys. B **514**, 245 (1998) [arXiv:hep-ph/9707223].
- [35] R. Barbieri, M. Caffo, R. Gatto and E. Remiddi, Phys. Lett. B **95**, 93 (1980), Nucl. Phys. B **192**, 61 (1981).
- [36] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
- [37] N. Brambilla *et al.* [Quarkonium Working Group], arXiv:hep-ph/0412158.
- [38] V. G. Kartvelishvili, A. K. Likhoded, and V. A. Petrov Phys. Lett. B **78** (1978) 615
- [39] M. Ablikim, M. N. Achasov, and L. An, *et al* [BESIII Collaboration] arXiv:hep-ph/1104.5068.